RADIO SCIENCE Journal of Research NBS/USNC-URSI
Vol. 69D, No. 2, February 1965

Insulated and Loaded Loop Antenna Immersed in a Conducting Medium[1]

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(Received August 20, 1964)

Previous analyses of the driving-point impedance of a small loop in an insulating radome immersed in a conducting medium are extended to include the effect of a permeable core loading the loop. It is shown that the effective area of the loop is increased by a gain factor previously derived for the loaded loop in free space. The effects of the conducting medium in this factor are negligible.

1. Introduction

The insulated electric loop antenna located in a dissipative medium was first analyzed by Wait [1952] who approximated the loop by an infinitesimal magnetic dipole located at the center of a spherical radome. Later, Wait [1957] reinvestigated this problem showing that, if the radius of the insulating radome is small compared with a skin depth in the dissipative medium, then essentially the same results are obtained for a finite sized loop in the radome. In this paper the problem of a small loop in an insulating radome is reinvestigated to determine the effects caused by loading the loop with a permeable material.

[1] This work was sponsored under Contract Nonr 2798(01) (FBM).

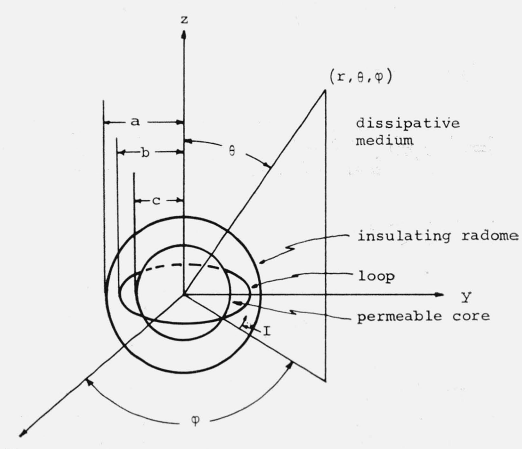

FIGURE 1. *Loaded loop antenna in a spherical insulating cavity.*

The surrounding dissipative medium is infinite in extent.

The spherical coordinates (r, θ, ϕ) are used. The insulating sphere has a radius $r=a$ which is small compared to a skin depth in the dissipative medium. The loop is in the $\theta=\pi/2$ plane and has a radius $b \leq a$. The current in the loop is assumed to be uniform. The permeable core is spherical with a radius $c \leq b$. The relative permeability of the core is designated by $\mu_r=\mu/\mu_0$. (See fig. 1.)

2. Formal Solution

A method of analysis is used which is parallel to that used by Wait [1957]. This problem has polar symmetry and may be described by the scalar potential A such that

$$E_\varphi = -j\omega\mu A,$$

$$H_r = \frac{1}{r \sin\theta}\frac{\partial}{\partial\theta}(A \sin\theta),$$

$$H_\theta = \frac{-1}{r}\frac{\partial}{\partial r}(rA). \tag{1}$$

The other field components are zero.

Since the loop is small and is assumed to have a uniform current distribution, the primary field of the loop is given by [Wait, 1957]:

$$rA^p = \frac{Ib}{2}\sum_{n=1}^{\infty}\frac{P_n^1(0)}{n(n+1)}P_n^1(\cos\theta)\begin{cases}\left(\dfrac{b}{r}\right)^n, & b<r<\infty \\ \left(\dfrac{r}{b}\right)^{n+1}, & 0<r<b\end{cases} \tag{2}$$

The following potentials are introduced to take into account the effects of the permeable core and the dissipative medium. When $a \leq r < \infty$,

$$rA = \frac{Ib}{2} \sum_{n=1}^{\infty} \frac{P_n^1(0)}{n(n+1)} T_n^+ P_n^1(\cos\theta) \frac{k_n(\gamma r)}{k_n(\gamma a)}, \tag{3}$$

where $\gamma = (j\omega\mu\sigma + \omega^2\mu\epsilon)^{1/2}$, the sea water parameters, and where $k_n(z)$ is a spherical Hankel function defined by

$$k_n(z) = e^{-z} \sum_{m=0}^{n} \frac{(n+m)!}{m!(n-m)!(2z)^m}. \tag{4}$$

When $c < r < a$,

$$A = A^p + A^+ + A^-, \tag{5a}$$

where

$$rA^+ = \frac{Ib}{2} \sum_{n=1}^{\infty} \frac{P_n^1(0)}{n(n+1)} S_n^+ P_n^1(\cos\theta) \left(\frac{b}{r}\right)^n, \tag{5b}$$

$$rA^- = \frac{Ib}{2} \sum_{n=1}^{\infty} \frac{P_n^1(0)}{n(n+1)} S_n^- P_n^1(\cos\theta) \left(\frac{r}{b}\right)^{n+1}. \tag{5c}$$

When $0 < r < c$,

$$rA = \frac{Ib}{2} \sum_{n=1}^{\infty} \frac{P_n^1(0)}{n(n+1)} C_n P_n^1(\cos\theta) \left(\frac{r}{b}\right)^{n+1}. \tag{6}$$

The continuity of E_φ, and of H_θ, is required when $r=a$ and $r=c$. Application of these boundary conditions determines the coefficients in (3), (5b and 5c), and (6):

$$S_n^- = S_n \left(\frac{1+K_n}{1-K_n S_n}\right), \tag{7a}$$

$$S_n^+ = K_n(1+S_n^-), \tag{7b}$$

$$T_n^+ = T_n \left(\frac{1+K_n}{1-K_n S_n}\right), \tag{7c}$$

$$C_n = \frac{(2n+1)(1+S_n^-)}{n(\mu_r+1)+1}. \tag{7d}$$

Where,

$$K_n = \left[\frac{(n+1)(\mu_r-1)}{n(\mu_r+1)+1}\right] \left(\frac{c}{b}\right)^{2n+1},$$

$$S_n = \left(\frac{n+\alpha_n}{n+1-\alpha_n}\right) \left(\frac{b}{a}\right)^{2n+1},$$

$$T_n = \left(\frac{2n+1}{n+1-\alpha_n}\right) \left(\frac{b}{a}\right)^n,$$

$$\alpha_n = (\gamma a) \frac{k_n'(\gamma a)}{k_n(\gamma a)}.$$

3. Discussion of Results

The electric field in the dissipative medium can be written by combining (1), (3), and (7c). The first term in the resultant series represents the contribution of the effective magnetic dipole source of the antenna under discussion. The other terms in the series represent the contribution of higher order multipoles which are either zero or small. Thus, the equivalent magnetic dipole moment is computed to be

$$m = SI(1+K_1)\left\{1 + \left[1 - 2K_1\left(\frac{b}{a}\right)^3\right]\frac{(\gamma a)^2}{6} + \ldots\right\}, \tag{8}$$

where $S = \pi b^2$, the area of the loop. Neglecting terms of the order of $(\gamma a)^2$ and higher, it is observed that the effective area of the loop is increased by the factor $(1+K_1)$. When $c=b$ it is observed that $(1+K_1)$ is the same as the gain factor computed by Wait [1953] for the receiving antenna. This, of course, is consistent with the reciprocity theorem. It is also of interest to note that K_1 decreases as $(c/b)^3$. This indicates the desirability of having the loop wound tightly on the core.

The driving-point impedance of the antenna is computed using

$$Z = \frac{-1}{I} \int_0^{2\pi} E_\varphi r d\varphi,$$

where $r=b$ and $\theta = \pi/2$. Thus,

$$Z = Z_0 + \Delta Z$$

where Z_0 is the driving-point impedance of the unloaded loop in free space, and

$$\Delta Z = \frac{j\omega\mu 2\pi b}{I} (A^+ + A^-).$$

Using (5b and 5c),

$$\Delta Z = j\omega\mu_0 \pi b \sum_{n=1}^{\infty} \frac{(P_n^1(0))^2}{n(n+1)} \left[\frac{S_n(1+2K_n)+K_n}{1-K_n S_n}\right]. \tag{9}$$

When $|\gamma a| << 1$ then it can be shown that

$$S_n \sim \frac{-(\gamma a)^2}{4n^2-1} \left(\frac{b}{a}\right)^{2n+1}. \tag{10}$$

Then using the approximation that $1/(1-z) \doteq 1+z$ if $|z|$ is small, and neglecting terms of the order of S_n^2 because they contribute a negligible part to the reactance of ΔZ, (9) becomes

$$\Delta Z \doteq j\omega\mu_0 \pi b \sum_{n=1}^{\infty} \frac{(P_n^1(0))^2}{n(n+1)} [S_n(1+K_n)^2 + K_n],$$

Further,

$$\Delta R = \frac{\omega^2 \mu_0^2 \sigma S^2}{6\pi a} \left[(1+K_1)^2 + \frac{9}{280}(1+K_3)^2 \left(\frac{b}{a}\right)^4 + \ldots \right], \tag{11a}$$

$$\Delta X = \frac{\omega \mu_0 \pi b}{2} \left[K_1 + \frac{3K_3}{8} + \frac{15}{128} K_5 + \ldots \right]. \tag{11b}$$

The expression $Z_0 + j\Delta X$ represents the inductive reactance of the loaded loop in free space. The term ΔR represents the effect of the dissipative medium external to the insulating sphere. Neglecting terms of $(b/a)^4$ and higher because they are small, it is observed that the effective area of the loop is again multiplied by the factor $(1+K_1)$. This result is consistent with (8).

It is remarked that when the permeable material is removed then $K_n = 0$. If one notes that the factor $[P_n^1(0)]^2$ was inadvertently left out of Wait's expression, then the expression for ΔR given here reduces to the expression for ΔZ given by Wait [1957, eq (15)].

It is concluded that the effective area of a small loop in an insulating radome is increased by the factor $(1+K_1)$ when the loop is loaded with a spherical permeable material. The external dissipative medium does not modify this result to any practical extent because the magnetic properties of the insulation and the dissipative medium are the same.

4. References

Wait, J. R. (1952), The magnetic dipole antenna immersed in a conducting medium, Proc. IRE **40,** No. 10, 1244–1245.

Wait, J. R. (1953), Receiving properties of a wire loop with a spheroidal core, Can. Jour. of Tech. **31,** No. 1, 9–14.

Wait, J. R. (1957), Insulated loop antenna immersed in a conducting medium, J. Res. NBS **59,** No. 2, 133–137 RP2781.

Additional Related Reference

Wait, J. R., and K. P. Spies (1964), A note on the insulated loops antenna immersed in a conducting medium, J. Res. Radio Sci. NBS/USNC-URSI 68D, No. 11, 1249–1250.

(Paper 69D2–465)